Coastal Engineering Technical Note

Clarification of Wave Heights from Wave Information Studies (WIS)

PURPOSE: To clarify the definition of wave height used in Wave Information Study (WIS) reports. The WIS has produced a major archive of wave information near U.S. coastlines. Results are summarized in a special series of WIS reports. They are also available through the Corps' Sea State Engineering and Analysis System (CETN-I-23) and NOAA's National Climatic Data Center.

BACKGROUND: The WIS information is developed from numerical wind and wave models. Wave heights are derived from a calculated spectrum of wave energy, in common with most modern wave hindcasting and measurement programs. This wave height is referred to as H_s or H_{m0} in the various WIS reports.

The s in H_s means "significant". The concept of a significant wave height was first introduced when sailors were asked to report the height of the larger, well-formed waves, and omit entirely the low and poorly formed waves as part of the synoptic weather reports from ships. Comparisons of early wave gage records with observations led to the conclusion that the wave height given by observers was approximately equal to the average height of the one-third highest individual waves and the notation $H_{1/3}$ was introduced. The notations H_s , $H_{1/3}$, and H_{m0} all represent "significant wave height." The notation H_s is considered to be general, whereas $H_{1/3}$ and H_{m0} refer to significant height estimates obtained by specific approaches, as discussed in the following paragraphs. Wave height parameters are also discussed in CETN-I-10.

As analog and digital records of waves became available, it was noted that the distribution of wave heights in a record could be approximated by a mathematical distribution called the Rayleigh distribution (Longuet-Higgins, 1952). The Rayleigh distribution is derived under the assumptions that sea surface elevations follow a Gaussian distribution and the sea wave energy is concentrated in a narrow band of frequencies. For a distribution like this, one can derive exactly the average of the highest one-third or the average of any other fraction of the waves in the distribution. Thus, the observational $H_{1/3}$ can be related to a well defined statistical quantity. In a hand analysis (and digital as well) of water level traces, individual waves may be defined, crests and troughs identified, and heights estimated and compiled. In such an analysis, $H_{1/3}$ can be readily defined.

The m0 in H_{m0} refers to the zero-order moment of a

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distribution. In this case, it is the zero moment about the origin of the distribution of spectral density, $S(f)$, by frequency, f. Mathematically this is:

$$m_0 = \sum_{f=0}^{\infty} f^0 \, S(f)$$

which is simply the sum of spectral density over all frequencies. The WIS wave models work with discretized frequency bands, typically 20 bands, so the zero-order moment is just the sum of spectral density in these bands. It was shown by Kinsman (1965) p. 338, that m_0 is the variance of the wave record. The Rayleigh distribution can be expressed in terms of the variance, (Ochi, 1982) which leads to the result that:

$$H_{m0} = 4.005 \; (m_0)^{1/2}$$

or the more familiar approximation;

$$H_{m0} = 4 \; (\sigma)^{1/2}$$

where σ represents the sea surface variance.

When irregular wave profiles are approximately sinusoidal in shape, $H_{1/3}$ and the various estimates of H_{m0} are equivalent to within about 5 percent (Neumann and Pierson, 1966, p. 351). As the depth decreases and waves shoal prior to breaking near the shore, they become nonlinear and peaked in shape rather than sinusoidal. Typically H_{m0} changes gradually whereas $H_{1/3}$ increases rapidly prior to breaking due to increased steepness of individual waves. The Rayleigh distribution remains a good model up to the beginning of the surf zone (just prior to breaking). A more refined alternative distribution for the heights of near breaking waves is discussed by Hughes and Borgman (1987).

The $H_{1/3}$ is more representative than H_{m0} of actual crest-to-trough wave heights and should be used in those applications where the effect of individual waves is more important than the average wave energy. An approximate relationship between the two heights as a function of water depth and wavelength is given in Figure 1 (from Thompson and Vincent, 1985). A 10 percent difference between the two parameters is possible when the depth d (in ft) is less than or equal to $0.32 \; T_p^2$, where T_p is the spectral peak period (period corresponding to the highest peak of the frequency spectrum). The steepness parameter, ϵ, is defined as:

$$\epsilon = 0.25 \, H_{m0} / L_p$$

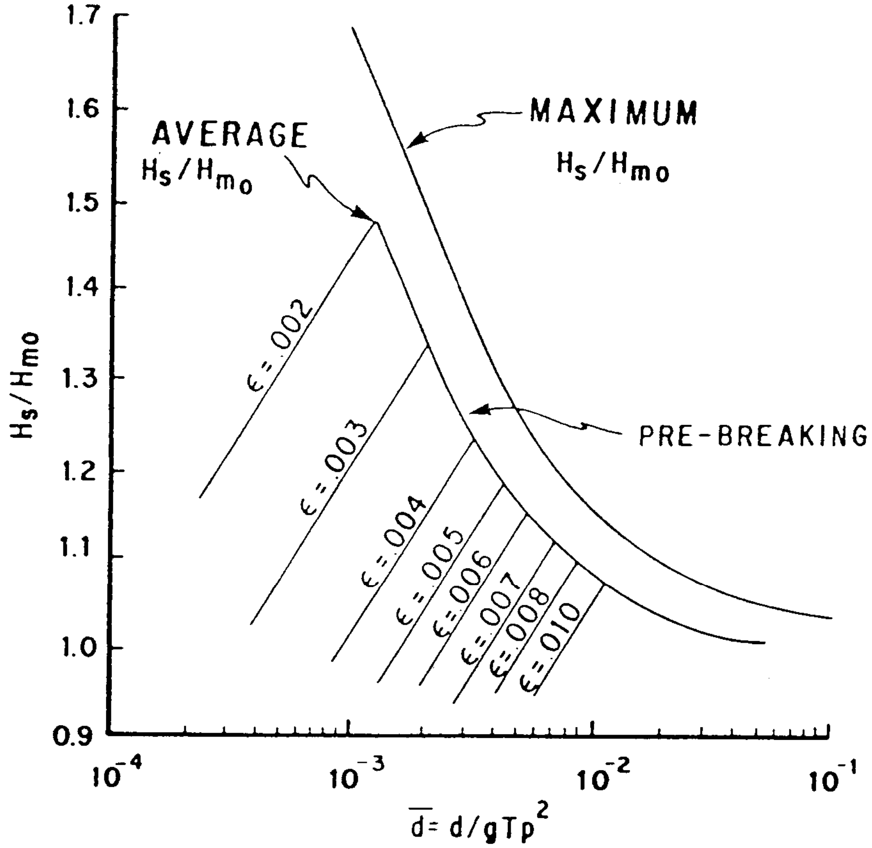

Figure 1. Variation of $H_{1/3}/H_{m0}$ as a function of relative depth $\bar{d}$ and significant steepness. (Both the average and maximum values of the ratio are given. The average value is read from the lower of the prebreaking line or the appropriate steepness line. Values of steepness intermediate to those plotted should be constructed as lines parallel to those plotted and spaced in between by linear interpolation. For values of steepness greater than 0.01, the ratio may be assumed to be 1.)

where L_p is the wavelength of a wave with period T_p in water depth d. Ten percent differences between $H_{1/3}$ and H_{m0} could be expected in depths of 32 ft (10 m) and 72 ft (22 m) for 10-s and 15-s periods, respectively. Larger discrepancies are possible for shallower depths up to the point of breaking. After breaking $H_{1/3}$ can be slightly less than H_{mo}.

WIS SIGNIFICANT WAVE HEIGHTS: The WIS significant wave heights reported from all Phase I and II models are energy-based, H_{m0}. Included are wave heights in WIS Reports 2, 6, 14, 15, 16, 18, and 19. Some of the WIS stations are in relatively shallow depth. For example, the shallowest station in the Atlantic and Gulf coast hurricane report is in a depth of 13 ft (4 m) (Appendix C, WIS Report 19, Abel et al, 1989). At this station,

$H_{1/3}$ could exceed H_{m0} by 20 and 35 percent for nonbreaking wave heights (i.e., heights less than approximately 0.78 * 13 ft, or approximately 10 ft) and 10- and 15-s periods, respectively. Differences such as these could be important for some design considerations.

The WIS significant heights from Phase III (WIS reports 9 and 17) are produced under a number of simplifying assumptions and because of this the distinction between $H_{1/3}$ and H_{m0} is generally less of a concern.

SUMMARY: Significant wave height information in all WIS Phase I and II reports is an energy-based parameter, H_{m0}. The more traditional wave height-based parameter, $H_{1/3}$ is more appropriate for some design considerations. It can be estimated from H_{m0} using Figure 1 if water depth and peak wave period are also known.

ADDITIONAL INFORMATION: For more information, contact Dr. Bob Jensen Robert.E.Jensen@usace.army.mil of the Coastal and Hydraulics Laboratory.

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